Communicating Mathematics

Read and study the lesson to answer each question.

1. **Describe** the conditions where the Law of Sines indicates that a triangle cannot exist.
2. **Draw** two triangles where \( A = 30^\circ, a = 6, \) and \( b = 10. \) Calculate and label the degree measure of each angle rounded to the nearest tenth.
3. **Write** the steps needed to solve a triangle if \( A = 120^\circ, a = 28, \) and \( b = 17. \)

Guided Practice

Determine the number of possible solutions for each triangle.

4. \( A = 113^\circ, a = 15, b = 8 \)
5. \( B = 44^\circ, a = 23, b = 12 \)

Find all solutions for each triangle. If no solutions exist, write none. Round to the nearest tenth.

6. \( C = 17^\circ, a = 10, c = 11 \)
7. \( A = 140^\circ, b = 10, a = 3 \)
8. \( A = 38^\circ, b = 10, a = 8 \)
9. \( C = 130^\circ, c = 17, b = 5 \)

10. **Communications** A vertical radio tower is located on the top of a hill that has an angle of elevation of \( 10^\circ. \) A 70-foot guy wire is attached to the tower 45 feet above the hill.
   a. Make a drawing to illustrate the situation.
   b. What angle does the guy wire make with the side of the hill?
   c. How far from the base of the tower is the guy wire anchored to the hill?

Exercises

Determine the number of possible solutions for each triangle.

11. \( A = 57^\circ, a = 11, b = 19 \)
12. \( A = 30^\circ, a = 13, c = 26 \)
13. \( B = 61^\circ, a = 12, b = 8 \)
14. \( A = 58^\circ, C = 94^\circ, b = 17 \)
15. \( C = 100^\circ, a = 18, c = 15 \)
16. \( B = 37^\circ, a = 32, b = 27 \)
17. If \( A = 65^\circ, a = 55, \) and \( b = 57, \) how many possible values are there for \( B? \)

Find all solutions for each triangle. If no solutions exist, write none. Round to the nearest tenth.

18. \( a = 6, b = 8, A = 150^\circ \)
19. \( a = 26, b = 29, A = 58^\circ \)
20. \( A = 30^\circ, a = 4, b = 8 \)
21. \( C = 70^\circ, c = 24, a = 25 \)
22. \( A = 40^\circ, B = 60^\circ, c = 20 \)
23. \( a = 14, b = 12, B = 90^\circ \)
24. \( B = 36^\circ, b = 19, c = 30 \)
25. \( A = 107.2^\circ, a = 17.2, c = 12.2 \)
26. \( A = 76^\circ, a = 5, b = 20 \)
27. \( C = 47^\circ, a = 10, c = 16 \)
28. \( B = 40^\circ, b = 42, c = 60 \)
29. \( b = 40, a = 32, A = 125.3^\circ \)
30. Copy the triangle at the right and label all measurements of the triangle.
31. Find the perimeter of each of the two noncongruent triangles where \( a = 15 \), \( b = 20 \) and \( A = 29^\circ \).

32. There are two noncongruent triangles where \( B = 55^\circ \), \( a = 15 \), and \( b = 13 \). Find the measures of the angles of the triangle with the greater perimeter.

33. Gears An engineer designed three gears as shown at the right. What is the measure of \( \theta \)?

34. Critical Thinking If \( b = 14 \) and \( A = 30^\circ \), determine the possible values of \( a \) for each situation.
   - a. The triangle has no solutions.
   - b. The triangle has one solution.
   - c. The triangle has two solutions.

35. Architecture The original height of the Leaning Tower of Pisa was \( 184 \frac{1}{2} \) feet. At a distance of 140 feet from the base of the tower, the angle of elevation from the ground to the top of the tower is \( 59^\circ \). How far is the tower leaning from the original vertical position?

36. Navigation The captain of the Coast Guard Cutter Pendant plans to sail to a port that is 450 miles away and \( 12^\circ \) east of north. The captain first sails the ship due north to check a buoy. He then turns the ship and sails 316 miles to the port.
   - a. In what direction should the captain turn the ship to arrive at the port?
   - b. How many hours will it take to arrive at the turning point if the captain chooses a speed of 23 miles per hour?
   - c. Instead of the plan above, the captain decides to sail 200 miles north, turn through an angle of \( 20^\circ \) east of north, and then sail along a straight course. Will the ship reach the port by following this plan?

37. Communications A satellite is orbiting Earth every 2 hours. The satellite is directly over a tracking station which has its antenna aimed \( 45^\circ \) above the horizon. The satellite is orbiting 1240 miles above Earth, and the radius of Earth is about 3960 miles. How long ago did the satellite pass through the beam of the antenna? (Hint: First calculate \( \theta \).)