5-2 Study Guide and Intervention

Verifying Trigonometric Identities

Verify Trigonometric Identities To verify an identity means to prove that both sides of the equation are equal for all values of the variable for which both sides are defined.

Example: Verify that \( \frac{\sec^2 x - 1}{\sec^2 x} = \sin^2 x \).

The left-hand side of this identity is more complicated, so start with that expression first.

\[
\frac{\sec^2 x - 1}{\sec^2 x} = \frac{(\tan^2 x + 1) - 1}{\sec^2 x} = \frac{\tan^2 x}{\sec^2 x} = \frac{\sin^2 x}{\cos^2 x} \cdot \cos^2 x = \sin^2 x \cdot \cos^2 x = \sin^2 x
\]

Notice that the verification ends with the expression on the other side of the identity.

Exercises

Verify each identity.

1. \( \sec \theta - \cos \theta = \sin \theta \tan \theta \)

\[
\sec \theta - \cos \theta = \frac{1}{\cos \theta} - \cos \theta = \frac{1 - \cos^2 \theta}{\cos \theta} = \frac{\sin^2 \theta}{\cos \theta} = \sin \theta \left(\frac{\sin \theta}{\cos \theta}\right) = \sin \theta \tan \theta
\]

2. \( \sec \theta = \sin (\theta + \cot \theta) \)

\[
\sin \theta (\tan \theta + \cot \theta) = \sin \theta \left(\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}\right) = \sin \theta \left(\frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta}\right) = \sin \theta \left(\frac{1}{\cos \theta \sin \theta}\right) = \frac{1}{\cos \theta} = \sec \theta
\]

3. \( \tan \theta \csc \theta \cos \theta = 1 \)

\[
\tan \theta \csc \theta \cos \theta = \left(\frac{\sin \theta}{\cos \theta}\right) \left(\frac{1}{\sin \theta}\right) \cos \theta = 1
\]

4. \( \frac{\csc^2 \theta - \cot^2 \theta}{1 - \sin^2 \theta} = \sec^2 \theta \)

\[
\frac{\csc^2 \theta - \cot^2 \theta}{1 - \sin^2 \theta} = \frac{(\cot^2 \theta + 1) - \cot^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta} = \sec^2 \theta
\]
5-2 Study Guide and Intervention (continued)

Verifying Trigonometric Identities

Identifying Identities and Nonidentities You can use a graphing calculator to test whether an equation might be an identity by graphing the functions related to each side of the equation. If the graphs of the related functions do not coincide for all values of \( x \) for which both functions are defined, the equation is not an identity. If the graphs appear to coincide, you can verify that the equation is an identity by using trigonometric properties and algebraic techniques.

Example: Use a graphing calculator to test whether \( \csc \theta - \sin \theta = \cot \theta \cos \theta \) is an identity. If it appears to be an identity, verify it. If not, find an \( x \)-value for which both sides are defined but not equal.

The equation appears to be an identity because the graphs of the related functions coincide. Verify this algebraically.

\[
csc \theta - \sin \theta = \frac{1}{\sin \theta} - \sin \theta
\]

Rewrite in terms of sine using a Reciprocal Identity.

\[
= \frac{1 - \sin^2 \theta}{\sin \theta}
\]

Rewrite using a common denominator.

\[
= \frac{\cos^2 \theta}{\sin \theta}
\]

Pythagorean Identity

\[
= \frac{\cos \theta}{\sin \theta} \cdot \cos \theta
\]

Factor \( \cos^2 \theta \).

\[
= \cot \theta \cos \theta \checkmark
\]

Rewrite in terms of \( \cot \theta \) using a Quotient Identity.

Exercises

Test whether each equation is an identity by graphing. If it appears to be an identity, verify it. If not, find an \( x \)-value for which both sides are defined but not equal.

1. \( \sin x + \cos x \cot x = \csc x \)

\[
\sin x + \cos x \cot x
\]

Rewrite in terms of sine using a Reciprocal Identity.

\[
= \sin x + \cos x \left( \frac{\cos x}{\sin x} \right)
\]

Rewrite using a common denominator.

\[
= \sin x + \frac{\cos^2 x}{\sin x}
\]

Pythagorean Identity

\[
= \frac{1}{\sin x} = \csc x
\]