5-1 Study Guide and Intervention
Trigonometric Identities

Basic Trigonometric Identities An equation is an identity if the left side is equal to the right side for all values of the variable for which both sides are defined. Trigonometric identities are identities that involve trigonometric functions.

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<th>Reciprocal Identities</th>
<th>Pythagorean Identities</th>
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<tr>
<td>(\sin \theta = \frac{1}{\csc \theta})</td>
<td>(\sin^2 \theta + \cos^2 \theta = 1)</td>
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<tr>
<td>(\csc \theta = \frac{1}{\sin \theta})</td>
<td>(\tan^2 \theta + 1 = \sec^2 \theta)</td>
</tr>
<tr>
<td>(\cos \theta = \frac{1}{\sec \theta})</td>
<td>(\cot^2 \theta + 1 = \csc^2 \theta)</td>
</tr>
<tr>
<td>(\tan \theta = \frac{1}{\cot \theta})</td>
<td></td>
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</table>

Example: If \(\sin \theta = \frac{3}{5}\) and \(0^\circ < \theta < 90^\circ\), find \(\tan \theta\).

Use two identities to relate \(\sin \theta\) and \(\tan \theta\).

\[\sin^2 \theta + \cos^2 \theta = 1 \quad \text{(Pythagorean Identity)}\]

\[\left(\frac{3}{5}\right)^2 + \cos^2 \theta = 1 \quad \sin \theta = \frac{3}{5}\]

\[\cos^2 \theta = \frac{16}{25} \quad \text{Simplify.}\]

\[\cos \theta = \pm \sqrt{\frac{16}{25}} \text{ or } \pm \frac{4}{5}\]

Take the square root of each side.

Since \(0^\circ < \theta < 90^\circ\), \(\cos \theta\) is positive.

Thus, \(\cos \theta = \frac{4}{5}\).

Now find \(\tan \theta\).

\[\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \text{(Quotient identity)}\]

\[\tan \theta = \frac{\frac{3}{5}}{\frac{4}{5}} \quad \sin \theta = \frac{3}{5}, \cos \theta = \frac{4}{5}\]

\[\tan \theta = \frac{3}{4} \quad \text{Simplify.}\]

Exercises

Find the value of each expression using the given information.

1. If \(\cot \theta = \frac{12}{5}\), find \(\tan \theta\). \(\frac{5}{12}\)

2. If \(\sin \theta = -\frac{1}{4}\), find \(\csc \theta\). \(-4\)

3. If \(\tan \alpha = \frac{2}{3}\), find \(\cot \alpha\). \(\frac{3}{2}\)

4. If \(\sec \beta = -2\), find \(\csc \left(\beta - \frac{\pi}{2}\right)\). \(2\)

5. If \(\cot \alpha = -\frac{4}{3}\) and \(\sin \alpha < 0\), find \(\cos \alpha\) and \(\csc \alpha\). \(\cos \alpha = \frac{4}{5}, \text{ and } \csc \alpha = -\frac{5}{3}\)

6. If \(\sec \alpha = -4\) and \(\csc \alpha > 0\), find \(\cos \alpha\) and \(\tan \alpha\). \(\cos \alpha = -\frac{1}{4}, \text{ and } \tan \alpha = -\sqrt{15}\)
5-1 Study Guide and Intervention (continued)

Trigonometric Identities

Simplify and Rewrite Trigonometric Expressions You can apply trigonometric identities and algebraic techniques such as substitution, factoring, and simplifying fractions to simplify and rewrite trigonometric expressions.

Example: Simplify each expression.

a. \( \sec x - \cos x \)

\[
\sec x - \cos x = \frac{1}{\cos x} - \cos x
\]
\[
= \frac{1-\cos^2 x}{\cos x}
\]
\[
= \frac{\sin^2 x}{\cos x}
\]
\[
= \sin x \left( \frac{\sin x}{\cos x} \right)
\]
\[
= \sin x \tan x
\]

Reciprocal Identity

Add.

Pythagorean Identity

Factor.

Quotient Identity

b. \( \csc x \cot^2 x + \frac{1}{\sin x} \)

\[
csc x \cot^2 x + \frac{1}{\sin x} = \csc x \cot^2 x + \csc x
\]
\[
= \csc x (\csc^2 x - 1) + \csc x
\]
\[
= \csc^3 x - \csc x + \csc x
\]
\[
= \csc^3 x
\]

Reciprocal Identity

Pythagorean Identity

Distributive Property

Simplify.

Exercises

Simplify each expression.

1. \( \cos x (\tan x + \cot x) \)  \( \csc x \)

2. \( \sin x + \cos x \cot x \)  \( \csc x \)

3. \( \frac{\csc^2 x}{1 + \tan^2 x} \)  \( \cot^2 x \)

4. \( (\sec x - \tan x)(\csc x + 1) \)  \( \cot x \)

5. \( (\cot^2 x + 1)(\sec^2 x - 1) \)  \( \sec^2 x \)

6. \( 1 + \frac{\tan^2 x}{1 + \sec x} \)  \( \sec x \)

7. \( \csc x \sin x + \cot^2 x \)  \( \csc^2 x \)

8. \( \cos x (1 + \tan^2 x) \)  \( \sec x \)

9. \( \frac{\cos \left( \frac{\pi}{2} - x \right)}{\csc x} \)  \( \sin^2 x \)

10. \( \frac{\cos \left( \frac{\pi}{2} - x \right)}{\csc x} + \cos^2 x \)  \( 1 \)