NAME _____ DATE _____ PERIOD _____

4-7

Study Guide

Radical Equations and Inequalities

Equations in which radical expressions include variables are known as **radical equations.** To solve radical equations, first isolate the radical on one side of the equation. Then raise each side of the equation to the proper power to eliminate the radical expression. This process of raising each side of an equation to a power often introduces **extraneous solutions.** Therefore, it is important to check all possible solutions in the original equation to determine if any of them should be eliminated from the solution set. **Radical inequalities** are solved using the same techniques used for solving radical equations.

Example 1 **Solve $3 = \sqrt[3]{x^2 - 2x + 1} - 1$.**

$$\begin{aligned} 3 &= \sqrt[3]{x^2 - 2x + 1} - 1 \\ 4 &= \sqrt[3]{x^2 - 2x + 1} && \textit{Isolate the cube root.} \\ 64 &= x^2 - 2x + 1 && \textit{Cube each side.} \\ 0 &= x^2 - 2x - 63 \\ 0 &= (x - 9)(x + 7) && \textit{Factor.} \end{aligned}$$

$x - 9 = 0$ $\qquad$ $x + 7 = 0$

$x = 9$ $\qquad$ $x = -7$

Check both solutions to make sure they are not extraneous.

$x = 9$: $3 = \sqrt[3]{x^2 - 2x + 1} - 1$

$3 \stackrel{?}{=} \sqrt[3]{(9)^2 - 2(9) + 1} - 1$

$3 \stackrel{?}{=} \sqrt[3]{64} - 1$

$3 \stackrel{?}{=} 4 - 1$

$3 = 3$ ✔

$x = -7$: $3 = \sqrt[3]{x^2 - 2x + 1} - 1$

$3 \stackrel{?}{=} \sqrt[3]{(-7)^2 - 2(-7) + 1} - 1$

$3 \stackrel{?}{=} \sqrt[3]{64} - 1$

$3 \stackrel{?}{=} 4 - 1$

$3 = 3$ ✔

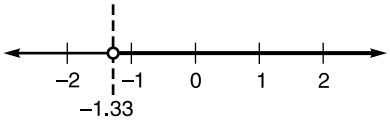
Example 2 **Solve $2\sqrt{3x + 5} > 2$.**

$$\begin{aligned} 2\sqrt{3x + 5} &> 2 \\ 4(3x + 5) &> 4 && \textit{Square each side.} \\ 3x + 5 &> 1 && \textit{Divide each side by 4.} \\ 3x &> -4 \\ x &> -1.33 \end{aligned}$$

In order for $\sqrt{3x + 5}$ to be a real number, $3x + 5$ must be greater than or equal to zero.

$$\begin{aligned} 3x + 5 &\geq 0 \\ 3x &\geq -5 \\ x &\geq -1.67 \end{aligned}$$

Since -1.33 is greater than -1.67, the solution is $x > -1.33$. Check this solution by testing values in the intervals defined by the solution. Then graph the solution on a number line.



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4-7

Practice

Radical Equations and Inequalities

Solve each equation.

1. $\sqrt{x-2} = 6$

2. $\sqrt[3]{x^2-1} = 3$

3. $\sqrt[3]{7r+5} = -3$

4. $\sqrt{6x+12} - \sqrt{4x+9} = 1$

5. $\sqrt{x-3} - 3\sqrt{x+12} = -11$

6. $\sqrt{6n-3} = \sqrt{4+7n}$

7. $5 + 2x = \sqrt{x^2-2x+1}$

8. $3 - \sqrt{r+1} = \sqrt{4-r}$

Solve each inequality.

9. $\sqrt{3r+5} > 1$

10. $\sqrt{2t-3} < 5$

11. $\sqrt{2m+3} > 5$

12. $\sqrt{3x+5} < 9$

13. ***Engineering*** A team of engineers must design a fuel tank in the shape of a cone. The surface area of a cone (excluding the base) is given by the formula $S = \pi\sqrt{r^2+h^2}$. Find the radius of a cone with a height of 21 meters and a surface area of 155 meters squared.

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4-7

Practice

Radical Equations and Inequalities

Solve each equation.

1. $\sqrt{x-2}=6$
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2. $\sqrt[3]{x^2-1}=3$
   $\pm 2\sqrt{7}$

3. $\sqrt[3]{7r+5}=-3$
   $-\frac{32}{7}$

4. $\sqrt{6x+12}-\sqrt{4x+9}=1$
   4

5. $\sqrt{x-3}-3\sqrt{x+12}=-11$
   4, $\frac{97}{16}$

6. $\sqrt{6n-3}=\sqrt{4+7n}$
   no real solution

7. $5+2x=\sqrt{x^2-2x+1}$
   $-\frac{4}{3}$

8. $3-\sqrt{r+1}=\sqrt{4-r}$
   0, 3

Solve each inequality.

9. $\sqrt{3r+5}>1$
   $r>-\frac{4}{3}$

10. $\sqrt{2t-3}<5$
    $\frac{3}{2}<t<14$

11. $\sqrt{2m+3}>5$
    $m>11$

12. $\sqrt{3x+5}<9$
    $-\frac{5}{3}<x<\frac{76}{3}$

13. ***Engineering*** A team of engineers must design a fuel tank in the shape of a cone. The surface area of a cone (excluding the base) is given by the formula $S=\pi\sqrt{r^2+h^2}$. Find the radius of a cone with a height of 21 meters and a surface area of 155 meters squared. **about 2.34 m**

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4-8

Practice

Modeling Real-World Data with Polynomial Functions

Write a polynomial function to model each set of data.

1. The farther a planet is from the Sun, the longer it takes to complete an orbit.

Distance (AU)	0.39	0.72	1.00	1.49	5.19	9.51	19.1	30.0	39.3
Period (days)	88	225	365	687	4344	10,775	30,681	60,267	90,582

Source: *Astronomy: Fundamentals and Frontiers*, by Jastrow, Robert, and Malcolm H. Thompson.

Sample answer: $f(x)=35x^2+962x-791$

2. The amount of food energy produced by farms increases as more energy is expended. The following table shows the amount of energy produced and the amount of energy expended to produce the food.

Energy Input (Calories)	606	970	1121	1227	1318	1455	1636	2030	2182	2242
Energy Output (Calories)	133	144	148	157	171	175	187	193	198	198

Source: *NSTA Energy-Environment Source Book.*

Sample answer: $f(x)=-3.9x^3+1.5x^2-0.1x+167.0$

3. The temperature of Earth's atmosphere varies with altitude.

Altitude (km)	0	10	20	30	40	50	60	70	80	90
Temperature (K)	293	228	217	235	254	269	244	207	178	178

Source: *Living in the Environment*, by Miller G. Tyler.

Sample answer: $f(x)=-0.0008x^3+0.1x^2-3.6x+274.7$

4. Water quality varies with the season. This table shows the average hardness (amount of dissolved minerals) of water in the Missouri River measured at Kansas City, Missouri.

Month	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Hardness ($CaCO_3$ ppm)	310	250	180	175	230	175	170	180	210	230	295	300

Source: *The Encyclopedia of Environmental Science*, 1974.

Sample answer: $f(x)=0.1x^4-1.6x^3+19.7x^2-110.0x+397.7$